Quiz 10

February 24, 2017

Show all work and circle your final answer.

1. If $y = e^{2x}$ $\rightarrow x = \frac{1}{2}\ln y$, *set up* (but do not evaluate) the integral for the area of the surface obtained by rotating the curve from $x = 0$ to $x = 1$ about the

   (a) x-axis

   $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1+(2e^{2x})^2}\,dx$ OR $= \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy = \sqrt{1+\left(\frac{1}{2y}\right)^2}\,dy$

   $A = \int_a^b 2\pi y\,ds$

   $= \boxed{\int_{x=0}^{x=1} 2\pi(\underbrace{e^{2x}}_{y})\sqrt{1+(2e^{2x})^2}\,dx}$ OR $= \boxed{\int_{y=1}^{y=e^2} 2\pi y\sqrt{1+\left(\frac{1}{2y}\right)^2}\,dy}$

   (b) y-axis

   ds is the same

   $A = \int_a^b 2\pi x\,ds$

   $= \boxed{\int_{x=0}^{x=1} 2\pi x\sqrt{1+(2e^{2x})^2}\,dx}$ OR $= \boxed{\int_{y=1}^{y=e^2} 2\pi(\underbrace{\tfrac{1}{2}\ln y}_{x})\sqrt{1+\left(\frac{1}{2y}\right)^2}\,dy}$

2. Find the exact length of the curve $y = 2 + 6x^{3/2}$ from $x = 0$ to $x = 1$.

   $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1+\left(6\cdot\frac{3}{2}x^{1/2}\right)^2}\,dx$

   $= \sqrt{1+81x}\,dx$

   $u = 1+81x$
   $du = 81\,dx$

   $L = \int_0^1 \sqrt{1+81x}\,dx$

   $= \int_{u=1}^{u=82} \frac{1}{81}u^{1/2}\,du$

   $= \frac{1}{81}\cdot\frac{2}{3}u^{3/2}\Big|_1^{82}$

   $= \boxed{\frac{2}{243}\left(82^{3/2}-1\right)}$

3. (a) What trig substitution would you use to evaluate $\displaystyle\int \frac{x}{\sqrt{a^2x^2+b^2}}\,dx$ for $a>0, b>0$?

(You may use the fact that $\sqrt{a^2x^2+b^2} = b\sqrt{(\frac{a}{b})^2x^2+1}$.)

$\boxed{\frac{a}{b}x = \tan\theta}$ or $\boxed{x = \frac{b}{a}\tan\theta}$

(b) Use your substitution to rewrite the above integral as a trigonometric integral. Do not evaluate.

$x = \frac{b}{a}\tan\theta$

$dx = \frac{b}{a}\sec^2\theta\, d\theta$

$$\int \frac{x}{b\sqrt{(\frac{a}{b}x)^2+1}}\,dx = \boxed{\int \frac{\frac{b}{a}\tan\theta}{b\sqrt{(\tan^2\theta)+1}} \cdot \frac{b}{a}\sec^2\theta\, d\theta}$$

$$= \frac{b}{a^2}\int \frac{\tan\theta\sec^2\theta}{\sec\theta}\,d\theta$$

$$= \frac{b}{a^2}\int \sec\theta\tan\theta\, d\theta$$